Simple analytical model of evapotranspiration in the presence of roots

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(Received 11 June 2014; revised manuscript received 20 September 2014; published 16 October 2014)

Evaporation of water out of a soil involves complicated and well-debated mechanisms. When plant roots are added into the soil, water transfer between the soil and the outside environment is even more complicated. Indeed, plants provide an additional process of water transfer. Water is pumped by the roots, channeled to the leaf surface, and released into the surrounding air by a process called transpiration. Prediction of the evapotranspiration of water over time in the presence of roots helps keep track of the amount of water that remains in the soil. Using a controlled visual setup of a two-dimensional model soil consisting of monodisperse glass beads, we perform experiments on actual roots grown under different relative humidity conditions. We record the total water mass loss in the medium and the position of the evaporating front that forms within the medium. We then develop a simple analytical model that predicts the position of the evaporating front as a function of time as well as the total amount of water that is lost from the medium due to the combined effects of evaporation and transpiration. The model is based on fundamental principles of evaporation fluxes and includes empirical assumptions on the quantity of open stomata in the leaves, where water transpiration occurs. Comparison between the model and experimental results shows excellent prediction of the position of the evaporating front as well as the total mass loss from evapotranspiration in the presence of roots. The model also provides a way to predict the lifetime of a plant.

DOI: 10.1103/PhysRevE.90.042716

PACS number(s): 87.19.rh, 68.03.Fg, 47.56.+r

I. INTRODUCTION

Evapotranspiration from a porous medium, a combined process of evaporation and transpiration, is an important water transport mechanism involved in soils containing living organisms such as plants. The diversity of the potential applications in which evapotranspiration plays an important role makes it a widely investigated case study of scientific and technological relevance. In a system consisting of a plant within a medium, evapotranspiration is a term that encompasses the two mechanisms by which water is transferred from the medium out of the system into the surrounding air. Evaporation, sometimes called drying, refers to the transfer of water from the medium to the air at the air-water interface. Transpiration refers to the transfer of water from the leaves into the air after absorption of water from the medium by the plant roots.

Water absorption by roots has already been modeled extensively. The earliest model on water uptake was developed by Philip [1] and Gardner [2]. It describes the extraction of water contained in a cylinder of soil around a root. Though simplistic, its main contribution analytically shows a decrease in water content around the root. Other models have shown that fluctuating water content affects root activity [3–5]. Subsequent models on water uptake, using both experimental results and numerical simulations [6–8] have been derived, introducing different empirical relations accounting for the complex nature of root activity. These empirical relations often couple water extraction and root growth. One example is the correlation between the profile of water content in the soil and the root architecture [6,9], showing increasing water saturation as a function of distance away from the root and also as a function of the root system topology.

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II. EXPERIMENTAL SETUP

Evaporation from a porous medium in the absence of roots has also been considerably studied. Several experimental and theoretical models [10,11] have described the challenges of predicting the evaporation behavior and the drying rates, due to the different mechanisms that are involved at the pore scale level, such as pore impregnation and drying [12], liquid flows due to pore size distribution [11], and corner flows due to grain contacts in the porous medium [13].

Although water evaporation and water absorption by roots have been studied separately, the aspect of evapotranspiration, where both processes occur simultaneously and are intricately coupled, has not been widely investigated due to the inherent complexity of the fluxes involved [3,14]. This research could reinvigorate important insights into water dynamics in a porous medium in the presence of roots and in the face of ongoing evaporation.

We present an analytical model describing the overall evapotranspiration of water out of a porous medium in the presence of live roots. This model is developed based on experimental results of water loss in the presence of roots under controlled humidity conditions. Using a two-dimensional (2D) visual experimental setup, the model shows the contribution of both evaporation and transpiration fluxes to overall water loss in the porous medium. This simple analytical model of evapotranspiration takes into account the availability of water in the roots vicinity. Results allow us to predict the total amount of water that has evaporated and transpired from the granular medium. It also gives us a way to estimate the lifetime of a plant, an important application in the agricultural industry.

Roots are challenging to study because they normally grow in opaque 3D environments [15] which evidently cannot easily be modeled in setups relying on visual measurements. Thus, 2D environments, also known as rhizotrons [9,16], are...
Hele-Shaw Cell (2D root growth cell)

FIG. 1. (Color online) A sample image of a lentil root grown inside a 2D Hele-Shaw cell of 10 cm × 15 cm. The effective dimensions of the cell are 7.5 cm × 13.5 cm × 1.2 mm.

commonly used due to their relative simplicity in terms of visualization [6,7,9,16,17] as well as their reproducibility. We perform root growth experiments in a 2D Hele-Shaw cell of size 10 cm × 15 cm, made of a pair of glass sheets. The sides of the cell are sealed with a commercial silicon paste and the upper portion remains open, allowing evaporation into the surrounding atmosphere. The inside of the cell measures 7.5 cm × 13.5 cm × 1.2 mm. It is filled with a monolayer of glass beads of diameter $d = 1 \pm 0.2$ mm (borosilicate, Sigma-Aldrich), thus the thickness, $e$, of the cell is roughly the diameter of the beads. In order to reduce wetting effects on the cell walls, the glass sheets are covered with a hydrophobic silane solution (OMS Chemicals). The glass beads, which are naturally hydrophilic, are simply washed with 0.1 M HCl and dried in an oven at 70°C overnight.

The 2D cell filled with glass beads is then fully saturated with water mixed with Hoagland nutrient solution (25% w/v). The nutrients are especially essential during the initial growth phase of the roots. The cell is fully immersed in a bath filled with the liquid, and we use a vacuum pump to saturate the pore spaces of the medium with water. The vacuum also helps remove unwanted bubbles.

We choose to use lentils, *Lens culinaris*, because they grow easily and their root system is relatively simple [16] and reproducible in its growth. In addition, the size of the lentil root is big enough to be clearly visualized with the naked eye. Lentil seeds are initially germinated for 2–3 days in a dark and moist environment. Once a radicle appears, we carefully select specimens of the same mass. The average radicle length after germination is $1.5 \pm 0.5$ cm. Each germinated lentil is placed in its own cell medium, and the cells are positioned beneath a white lamp used as a grow light under ambient conditions at temperature $T = 23 \pm 2°C$ and relative humidity $H_R = 45 \pm 5%$. The roots are then grown in preparation for the evapotranspiration experiments. Unlimited water and nutrients are provided during this phase, otherwise the plants quickly die. After 2–3 days, the roots have reached a more steady metabolic demand and the cells are ready for evapotranspiration experiments.

Each 2D cell is transferred to a controlled chamber (Model 5100 Electro-Tech Systems) which has a dual humidification and dehumidification system allowing for precise control of temperature and relative humidity. In its chamber, each 2D cell is placed on a scale to record its mass as a function of time. Images of the entire cell containing the root in its medium are taken at specific time intervals using a Canon 500D SLR camera with a 18- to 55-mm lens. A light source is used to back-illuminate the cells in order to effectively contrast and observe water distribution within the medium. A sample image of a 2D root growth cell is seen in Fig. 1, showing the lentil and its root growing in the medium.

Roots in granular materials typically adopt a curved profile. As a result, the contour length is measured using a segmentation method by creating small line segments. The total root contour length is measured as a summation of the lengths of the individual line segments. Preliminary tests of the segmentation method performed on various sets of curves of known length reveal a measurement error within ±5%.

### III. RESULTS AND DISCUSSION

Before discussing the effect of roots on the evapotranspiration process, let us briefly summarize the well-known features of gravity-dominated evaporation in a porous medium in the absence of roots. Several models and approaches have been discussed to describe and predict basic evaporation behavior. They include pore-network micromodels [18], capillary bundle models [10], and models using corner liquid films [19,20]. Evaporation, or drying, has been viewed as an immiscible fluid displacement process [21], where the saturating liquid in the medium is gradually removed from the pores [22]. Consider a medium initially fully saturated with liquid and allowed to evaporate. In the case of gravity dominated evaporation [10,13,19,22–26], the process exhibits two regimes. In the first regime, evaporation occurs at the liquid-air interface at the top surface of the sample. Below this surface, a layer called the partially saturated zone (PSZ), composed of an intricate network of liquid films and droplets, progressively develops [26,27]. Beneath this layer, the medium is fully saturated with water. The surface at the interface of the PSZ and the fully saturated zone is usually referred to as the percolation front, $z_{inf}$, because the morphology of the front can be described by percolation theory [26]. In the PSZ, the pores are connected via intricate interfaces, which serve as hydraulic capillary networks linking the percolation front to the top of the sample, where evaporation occurs. During the first regime, the liquid flow continues to support the evaporation demand and thus the drying rate is mainly determined by atmospheric conditions. The inset figure of Fig. 2(a) shows an image with the different zones in this first regime. The second regime starts as soon as capillary pressure can no longer sustain the rate of evaporation [24]. As a result, liquid films detach from the surface and at this point, a dry zone appears. The interface between the PSZ and the dry zone is called the evaporating...
FIG. 2. (Color online) (a) Typical evaporation curve in a 2D Hele-Shaw cell in the absence of roots: water mass loss is plotted as a function of time. In this figure, both regimes are represented by an image and the transition is indicated by the broken vertical line. (b) Comparison of water loss as a function of time in the presence (‡) and absence (−) of roots. The solid arrows (↘) point to the transition point between the first and second regimes. (c) Measured evaporating and percolating fronts denoted as \( z_{\text{sup}} \) and \( z_{\text{inf}} \), respectively, as a function of time. (d) Measured height of the PSZ, \( h \), where \( h = z_{\text{inf}} - z_{\text{sup}} \), as a function of time. The inset figures depict the evolution of the PSZ during regime 2, showing that its height \( h \) roughly remains constant. These curves were obtained under conditions of \( T = 32 \pm 2 \) °C and \( H_R = 20.0 \pm 2.0\% \).

Figure 2(a) depicts a typical evaporating curve in the absence of roots, showing both regimes as separated by the broken vertical line, which indicates the transition. As the process continues, both the evaporating and percolation fronts recede deeper into the sample while the thickness, \( h \), of the PSZ remains approximately constant, as seen in the series of inset figures in Fig. 2(d). Detailed mechanisms on the fundamentals of evaporation can be found in the literature, notably in Prat et al. [12,26,27], Yiotis et al. [13,23,28], Chauvet et al. [19,20], Lehmann et al. [10,25], and Shokri et al. [11,22].

Water loss in the presence of roots is made complicated by an additional process called transpiration. This process, inherent to plants, consists of the loss of water at the air-leaf interface [29,30]. It results from the opening and closing of stomata cells during their intake of carbon dioxide, which is essential for photosynthetic activity. These stomata cells are found at the surface of plant leaves. During this process of gas exchange, water vapor diffuses out of the stomata. The loss of water in the leaves induces a pressure differential across the plant, leading to a negative potential of water around the roots, which results in absorption of water by the roots to compensate for water loss and to be used in the plants own metabolism.

A schematic of the evapotranspiration process in the presence of roots is shown in Fig. 3(a), while some experimental images of the process in a typical 2D cell are shown in Fig. 3(b). The saturation of water in the medium is defined by \( \Phi \), which ranges from 0 to 1, 0 meaning the medium is completely dry and 1 denoting that it is completely saturated with water. The roots are initially exposed to water in the fully saturated zone, \( \Phi = 1 \). In the first regime, the evapotranspiration process creates a PSZ, where \( \Phi < 1 \), which is similar to the first regime of an evaporation-only process in the absence of roots. Over time, a dry zone, where \( \Phi = 0 \), develops (this is the second regime) and the entire root system is eventually completely buried within this dry zone and the plant dies (not shown). As shown in the last image in Fig. 3(b), we observe a depletion zone develop around the roots within the PSZ. To our knowledge, this zone has not been well studied but it appears to result from the root pumping activity. Water is still present in the form of films within this zone [2], which exhibits high resistance to water uptake [31], presumably because the films are smaller than in the rest of the PSZ. Our experiments show that the plant continues to live when the depletion zone has developed all around the root; it dies only when the root is well within the dry zone. This observation suggests that roots do extract water out of the depletion zone. For these reasons, we choose to consider the depletion zone as part of the PSZ in our experiments. In addition, our preliminary experiments of root length measurements from the beginning to the death of the plant show a minimal increase of less than 10%. Indeed, root growth rates are extremely slow compared to the rate of formation of the evaporating front. Thus, we choose to neglect root elongation in our analysis. Finally, our experiments also
FIG. 3. (Color online) (a) Schematic of the evapotranspiration process in the presence of roots. The saturation of water in the medium is defined by $\Phi$. The system starts out fully saturated, $\Phi = 1$. Water is lost from the system through evaporation and transpiration, leading to the appearance of a partially saturated zone (PSZ) of height $h = z_{\text{inf}} - z_{\text{sup}}$, which is a mixture of air and water ($\Phi < 1$). As water is lost during the first regime, a portion of the root’s length becomes exposed to the PSZ. The vertical distance between the tip of the root and the cell surface is $L_v$. During the second regime, a dry region ($\Phi = 0$) develops above the receding evaporating front, $z_{\text{sup}}$, which slows down evaporation. A depletion zone also develops, as illustrated by the small white area in the PSZ around the root. (b) Experimental images showing the evolution of the front during evapotranspiration in the presence of roots. The roots absorb water to replenish the amount of water lost by transpiration; this is clearly manifested by the appearance of a depletion zone around the roots. The depletion zone is considered part of the PSZ because thin films of water are still present in this zone.

show that the total mass of the plant increases by less than 5% throughout the experiments. Therefore, we choose to neglect the variations in plant mass.

We characterize water loss in the absence and in the presence of roots by measuring the total mass loss, the positions of the evaporating and percolating fronts, $z_{\text{sup}}$ and $z_{\text{inf}}$, respectively, and the average size height of the PSZ, $h$. Results are presented in the curves in Fig. 2.

Figure 2(b) shows water mass loss over time with and without roots. It is easily seen that the process results in higher mass loss when roots are present, showing that roots provide an additional pathway for water loss, namely transpiration. Note that the mass of the plant has not yet been subtracted from this curve. When we take into account the mass of the plant, which, as mentioned earlier, can be approximated as constant, the actual water loss in the presence of roots is still more pronounced. Given that all physical parameters (particle size, cell size, humidity, etc.) are similar and constant in both types of experiments, the difference in water uptake can clearly be attributed to the presence of roots.

Experimentally, the evaporating and percolation fronts develop faster in the presence of roots compared to without roots, as depicted in Fig. 2(c). This observation suggests faster water loss when roots are present. In addition, the percolation front, $z_{\text{inf}}$, approaches the same final value regardless of the presence of roots. The presence of roots, however, clearly affects the formation of the evaporating front, $z_{\text{sup}}$. This results in a clear difference in the PSZ height $h$, as shown in Fig. 2(d).

The final value of $h$ in the presence of roots is smaller than without roots. This is an interesting result as it provides a clue into the activity of the root within the PSZ. When more water is lost in the porous medium, this leads to faster formation of the receding front yet ultimately to a PSZ of smaller size.

Despite the minimal growth rate, the presence of roots clearly modifies the total evaporation dynamics of water out of a given porous medium. The next section describes a simple analytical model that predicts the total evapotranspiration flux emanating from the porous medium in the presence of roots. The model also provides an estimation of when the root will be completely in the dry zone, therefore predicting the lifetime of a plant.

IV. THEORETICAL CONSIDERATIONS

As can be seen from the experimental observations, two regimes of evapotranspiration are observed. There is a fast regime or regime 1, in which a connection of water exists between the percolating front and the top of the cell [10,11,13,26]. This allows for rapid flow of water out of the granular medium. The second regime (regime 2) appears when this connection is broken at the surface and as a result, an evaporating front occurs that recedes inside the medium at the same rate as the percolating front. A dry region develops above the evaporating front and below the surface. Evaporation
rates slow down as water now diffuses across this dry region. As water is simultaneously removed from the soil through evaporation and transpiration, we can define four different fluxes. $J_e$ and $J_t$ correspond to the evaporation flux during regimes 1 and 2, respectively, while $J_s$ and $J_p$ refer to the transpiration flux during regimes 1 and 2, respectively. If $m$ corresponds to the total water mass loss, then the rate at which the mass of water is taken out of the soil during each regime is

$$\frac{dm}{dt} = J_e + J_s + J_p,$$

where $S$ is the cross-sectional area of the surface. We choose to consider water mass loss as positive when water escapes from the soil in the upward direction. The subscripts “1” and “2” correspond to regimes 1 and 2, respectively, and are not valid simultaneously.

## A. Evaporation flux

In the first regime, liquid transport out of the cell proceeds via diffusion across an external transfer zone, $\delta$, and can be written using Fick’s law as follows:

$$J_e = D_e \left( \frac{C_{sat} - C_e}{\delta} \right),$$

where $D_e$ is the diffusion coefficient of water in the air, $C_{sat}$ is the concentration of water at the evaporation plane, and $C_e$ is the concentration of water in the surrounding atmosphere, which is directly related to the relative humidity, $H_R$. The value of $C_{sat}$ can be calculated from the saturation vapor pressure of water at a particular temperature using the ideal gas law.

The flux during the second regime $J_s$ can also be described via Fick’s law. Note that there are now two gradients building up in the system, resulting in a flux through the porous medium and another flux across the external mass transfer zone. The overall mass of water being conserved, these two fluxes are equal.

The flux of water through the porous medium can be written as follows:

$$J_p = D_p \left[ \frac{C_{sat} - C_s}{\delta_{st}} \right],$$

where $D_p$ is the diffusion coefficient of water across the porous medium, which is equal to $D_{st} \psi^{1.5}$ [32,33], $C_s$ is the concentration of water at the cell surface, the value of which is unknown. The flux of water from the cell surface to the surrounding atmosphere is derived as follows:

$$J_s = D_a \left[ \frac{C_{sat} - C_e}{\delta} \right].$$

By equating Eq. (3) and Eq. (4), we can derive an expression for $C_s$ in either Eq. (3) or Eq. (4) and obtain an expression for $J_e$,

$$J_e = \frac{C_{sat} - C_e}{\delta_{st}} \frac{\partial \psi}{\partial t} + \frac{\partial}{\partial t}.\frac{S_{st}}{S_{st}}$$

These expressions describe water transfer during regime 1 and regime 2 in a porous medium strictly in terms of evaporation and are thus valid in the absence and in the presence of roots.

## B. Transpiration flux

In this section, we address how the presence of roots modifies the overall water transfer dynamics due to the additional transpiration flux. A simple conceptual illustration of a leaf surface is shown in Fig. 4 and helps clarify the transpiration process. Transpiration occurs at the surface of plant leaves during photosynthesis when water vapor exits the leaves through the stomata, as carbon dioxide is taken in. The mechanisms regulating gas exchange in the stomata have been clearly described [29,30]. Plants have an internal regulation system that controls the opening and closing of the stomata. Transpiration is the main driving force for the movement of water from the root to the leaves.

We define the way water escapes out of the 2D medium by first imposing the condition that water must diffuse through the stoma in its vapor phase. Water travels from the interior of the leaf to the exterior across a distance $\delta_{st}$, which is the characteristic height of a stoma. Total flux density depends on the surface area $S_{st}$, which is the total surface area of the stomata. If the concentration of water inside the leaf is $C_{sat}$ and the concentration of water at the leaf surface is $C_s$, then by Fick’s law, the total mass of transpired water $m_t$ per unit time is

$$\frac{dm_t}{dt} = D_a \left( \frac{C_{sat} - C_s}{\delta_{st}} \right) S_{st}.$$  

Here we assume that the external mass transfer from the leaf surface remains constant. In reality, this is a complex problem as it is influenced by a multitude of parameters such as boundary layer thickness, atmospheric and environmental conditions, as well as surface area variations of the stomata. The transpiration is known to depend on where light strikes the leaves and on local wind speed [34].

All of the water diffusing through the pores also diffuses into the surrounding atmosphere of water concentration $C_s$. Thus, loss of mass per unit of time can also be written as follows:

$$\frac{dm_t}{dt} = D_a \left( \frac{C_s - C_s}{\delta} \right) S_{st}$$

where $S_{st}$ is the total surface area of the leaf and $\delta$ is the height of the external mass transfer zone at the leaf surface.

FIG. 4. (Color online) Illustration of a stoma at the leaf surface (adapted from Ref. [29]). Water vapor diffuses through the characteristic height $\delta$ of the stomata and across the external boundary layer $\delta$. $C_{sat}$ is the concentration of water inside the leaf, $C_s$ refers to the concentration of water at the surface of the leaf, and $C_e$ is the water concentration in the surrounding atmosphere. $C_e$ depends on the relative humidity.
By equating Eq. (6) and Eq. (7), we can solve for \( C_s \) and reinject its value in either equation to obtain an expression for transpiration,

\[
\frac{dm_t}{dt} = D_a \left[ \frac{\Delta C}{\delta_a + \delta (\frac{S}{A})} \right] S_a,
\]

where \( \Delta C = C_{sat} - C_e \). By introducing \( \phi_{st} \), which is defined as the fraction of leaf area occupied by stomata \( \phi_{st} = S_{st}/S_L \), we can then rewrite Eq. (8) as

\[
\frac{dm_t}{dt} = D_a \left( \frac{\Delta C}{\delta_a + \phi_{st} \delta} \right) \phi_{st} S_L.
\]

It is unclear how the area of the stomata, \( \phi_{st} \), varies. It is evident, however, that the stomata respond to external stimuli such as the quantity of water in the medium. So a dependence of \( \phi_{st} \) on water content is expected.

For the sake of simplicity, we make the following assumptions. First, during regime 1, when the evaporating front has not yet formed, thus \( z_{sup} = 0 \), the root is immersed in the PSZ. We assume that in this regime the stomata are open and \( \phi_{st} \) remains constant.

Second, during regime 2, the evaporating front has now formed and receded inside the medium, thus \( z_{sup} > 0 \), and part of the root system is now buried in the dry zone, while the remaining part is still immersed in the PSZ. The appearance of the dry zone results in the reduction of transpiration activity in order to conserve water inside the plant, and thus there is closing of the stomata. The simplest assumption that is feasible for the moment is that the surface of the stomata varies linearly with the height of the remaining PSZ in the medium that is exposed to the root, \( \phi_{st} \sim \beta (L_v - z_{sup}) \), where \( L_v \) is the vertical distance between the root tip and the topmost surface of the cell [see Fig. 3(a)] and \( \beta \) is a constant. Experimental results particularly on lentil roots in 2D models with granular media have shown a linear proportionality between the parameter \( L_v \) and the root length \( L_T \), as shown in Fig. 5.

Third, in Eq. (8) it is actually known that when the stomata are open, \( \phi_{st} \) varies between 0.1% and 1% depending on plants [29]. Since \( \delta \sim 1 \text{ mm} \), \( \delta \phi_{st} \) remains small compared to the height of the stomata, \( \delta_a \). Over time, this quantity decreases slightly. Therefore, we choose to neglect this variation in our model.

Fourth and last, we extend the second assumption by considering that there is also some relationship between the surface area of the leaf and the size of the root. Again, the simplest relationship is a linear proportionality, \( S_L = \alpha L_v \), where \( \alpha \sim 1.12 \times 10^{-2} \text{ m} \) is measured experimentally.

We can now obtain the expression for the transpiration flux in both regimes by using these four assumptions in Eq. (9), which becomes the following: During regime 1,

\[
J_1 = D_a \frac{\Delta C}{\delta_a + \phi_{st} \delta} \frac{\alpha \phi_{st} L_v}{S},
\]

while during regime 2,

\[
J_2(t) = D_a \frac{\Delta C}{\delta_a + \phi_{st} \delta} \frac{\alpha [L_v - z_{sup}(t)] L_v}{S},
\]

where \( S \) is the cross-sectional area at the cell surface.

As we saw earlier in Eq. (1), the total rate of water mass transferred out of the soil in the first regime is related to the evaporation and the transpiration fluxes by the following:

\[
\frac{dm}{dt} = J_e S + J_1 S.
\]

In the first regime, both the evaporation flux and the transpiration flux do not depend on time. Therefore the equation for the first regime can be readily integrated assuming that \( m(t = 0) = 0 \). Thus, the total mass of water that is extracted from the soil during regime 1 can be written as follows:

\[
m(t) = (J_e S + J_1 S) t.
\]

Equation (13) predicts that the mass varies linearly with time during regime 1. This result is consistent with experimental observations [see Fig. 2 and Fig. 6(b)].

In the second regime,

\[
\frac{dm}{dt} = J_e(t) S + J_2(t) S.
\]

At a certain time \( t \) during the experiment, the mass that has evaporated is the sum of the mass of water that was initially contained in the dry zone, \( z_{sup} \phi \rho S \), and the mass of water out of the partially saturated zone, \( h S \psi (1 - \Phi) \), where \( \psi \) is the porosity of the medium, \( \Phi \) the saturation of water in the medium, and \( \rho \) the density of water,

\[
m(t) = -z_{sup}(t) \psi \rho S + h S \psi (1 - \Phi).
\]

Using Eqs. (14) and (15), we obtain a differential equation for \( z_{sup} \). Let us call \( m_c \) the critical evaporated mass of water at which the system transitions from regime 1 to regime 2 at time \( t_c \). Thus, \( m(t_c) = m_c \), and at \( t = t_c \), \( z_{sup} = 0 \). We can then solve the differential equation for \( z_{sup}(t) \) using Mathematica and we obtain the following implicit expression:

\[
t = t_c + \frac{\rho \psi}{D_p \Delta C(2A/L_1)} \Pi,
\]

FIG. 5. Plot of root length, \( L_T \), as a function of distance between the root tip and cell surface, \( L_v \). These results reflect an average of the experiments performed for the characterization of lentil roots under fully saturated conditions. Experiments show a linear proportionality between root length and tip distance, as seen from the broken line.
FIG. 6. (Color online) (a) Position of the evaporating front $z_{\text{sup}}$ as a function of time in the presence of roots under various relative humidity conditions. The solid curves are calculated from the implicit expression for $z_{\text{sup}}$ shown in Eq. (16). The solid arrows point to the transition point between regime 1 and regime 2. (b) Total water mass loss curves in the porous medium in the presence of roots under different relative humidity conditions. The solid curves are generated using Eqs. (13), (15), and (16). The change in inflection of each curve, indicated by the solid arrows, denotes the transition point between regime 1 and regime 2. Again, our model fits experimental results.

where

$\Pi = F_2[\text{arcth}(F_3) - \text{arcth}(F_4)] + F_1 \ln(F_3), \quad \text{(17)}$

and

$F_1 = \sqrt{4 + \bar{A} (L_v + \bar{D})^2},$

$F_2 = 2 \sqrt{\bar{A}} (L_v + \bar{D}),$

$F_3 = \frac{\sqrt{\bar{A}} (\bar{D} - L_v + 2z_{\text{sup}})}{F_1},$

$F_4 = \frac{\sqrt{\bar{A}} (\bar{D} - L_v)}{F_1},$

$F_5 = \frac{1 + \bar{A} (L_v - z_{\text{sup}})(\bar{D} + z_{\text{sup}})}{1 + \bar{A} L_v \bar{D}}, \quad \text{(18)}$

Note that the model predicts the time at which $z_{\text{sup}} = L_v$, that is, when the entire root is buried in the dry zone and will imminently die. Thus, we can view Eq. (16) at $z_{\text{sup}} = L_v$ as an analytical expression of the plant’s lifetime.

C. Application of model to experiments

We now compare results derived from the model developed in the previous section to our experimental results. Values of several parameters defined earlier and used in the model are shown in Table I. We first apply the model to the evaporapotranspiration curves obtained in the presence of roots where the conditions are set to $H_R = 20.0 \pm 2.0\%$ and $T = 32 \pm 2^\circ C$. In these series of experiments, we record the position of the evaporating front, $z_{\text{sup}}$, and the total mass loss in the medium over time. The temperature value inside the chamber remains constant and root growth is limited and considered negligible.

In regime 1, water mass loss can be calculated using Eq. (13). In this regime, there is only one parameter to adjust: $\phi_{\text{st}}$, the fraction of leaf area occupied by open stomata. We find that the best fit is obtained with $\phi_{\text{st}} \approx 0.2\%$, which is consistent with what is reported in the literature [29].

In determining the transition time $t_c$ between regime 1 and regime 2, we use $m_c$, the water mass loss at $t_c$, as an adjustable parameter. The adjusted value of $m_c$ that we find is $m_c \sim 2$ g, which is of the right order of magnitude. Indeed, our preliminary experiments show that in the absence of roots, the water mass loss during regime 1 is approximately 3 g.

In regime 2, our model uses the approximation that $\phi_{\text{st}} \simeq \beta(L_v - z_{\text{sup}})$. Here $\beta$ is a constant and a fitting parameter that defines the proportionality relationship between the length of root within the PSZ and the surface area of the stomata. We find that the best fit is obtained with $\beta \approx 0.1$ m$^{-1}$.

Using these three fitting parameters, we then compare the model with the experimental data obtained at other humidity rates. The results are shown in Fig. 6. It is remarkable that by keeping all the fitting parameters constant but changing the relative humidity to $H_R = 40.0 \pm 2.0\%$ and $H_R = 75.0 \pm 2.0\%$, the model shows true predictive power.

Figure 6(a) shows the position of the evaporating front $z_{\text{sup}}$ as a function of time for different root systems grown in three...
different relative humidity conditions. The solid curves are calculated from the implicit expression in Eq. (16). The model shows considerable agreement with experimental data. Since $z_{\text{sup}}$ only begins to appear during regime 2 when it recedes inside the medium, the starting point of each curve reflects the transition time between the first and second regime of evaporation. In the figure, we can see that $z_{\text{sup}}$ recedes deeper at lower $H_R$ values.

Figure 6(b) shows water mass loss curves from evapotranspiration in the presence of roots as a function of time for the three different relative humidity rates. The solid curves are calculated using Eqs. (13), (15), and (16) for both regimes. Results show good agreement between the model and the experimental data. In this figure, lower $H_R$ denotes higher evaporation conditions and thus more mass of water is lost from the medium.

V. CONCLUSION

By basing our analysis on experimental results and fundamental principles of evapotranspiration flux, we have been able to establish a simple analytical model that estimates and predicts the total flux of water out of a soil in the presence of roots as well as the positions of the resulting evaporating fronts. This model takes into account two separate fluxes: the evaporation flux which occurs at the air-water interface and the transpiration flux which corresponds to the flux of water pumped by the plant roots and evaporated at the surface of leaves. Our model uses simple assumptions on the behavior of the plants in response to water stress. These assumptions include the dependence of the opening of stomata on the amount of root buried in the partially saturated zone. This model was confronted to experimental results. Using a simple model soil system, we have recorded the position of the evaporating front, $z_{\text{sup}}$ and the total mass loss curves resulting from the combined effect of evaporation and transpiration at three different humidity rates. We found that a set of fitting parameters, which are kept constant regardless of the value of the relative humidity, are sufficient to precisely predict both the quantity of water lost from the soil and the architecture of water distribution in the soil. From this model, we also infer an analytical expression that predicts the lifetime of a plant.

Water transport in plants is ensured by very complex mechanisms. Factors such as wind speed, carbon dioxide intake, and local variations in the amount of light, which were neglected in our model, likely affect evapotranspiration. However, we believe that our model can be used as a starting point for future investigations. The results presented here offer fresh insights into the quantification of water loss during the combined effects of evaporation and transpiration, thereby improving efforts to estimate plant’s lifetime—a key aspect in agriculture.

ACKNOWLEDGMENTS

The study is financially supported by Solvay, Inc. and Centre National de la Recherche Scientifique (CNRS). We thank everyone in the COMPASS (UMI 3254) laboratory and all those involved and who have contributed to the water retention project.


